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MATHEMATICAL MODELING OF THE WELL-BED SYSTEM UNDER THE GASLIFT OPERATION

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Abstract. Formulation of the mathematical model of a well-bed system is considered in this paper. It is proposed that the formation of gas liquid mixture area, i.e. the area between a lift shoe and bed has to be considered. On the basis of conducted gaslift well field research there was determined the influence of immersion depth on a gas-liquid mixture (GLM) formation and obtained generalized for all type of wells empirical expression for GLM formation process describing the dependence of the latter on an injection rate, lift diameter and immersion depth. Based on gas, GLM and fluid flow equations proposed in [4] a technique for gaslift process forecast was elaborated where the empirical expression is used as a boundary condition at the inlet point of injected gas.

Keywords: gaslift, well-bed system, submergence.

AMS Subject Classification: 97M50.

1. Introduction

It is known that gaslift (Fig. 1) is one of the most widely used mechanized and highly efficient process of oil well operation.

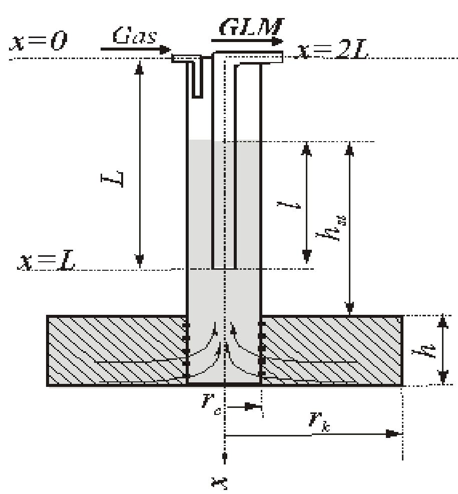

Fig. 1. Gaslift well.

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Thus, it should be noted, efficiency of this method depends on the correct definition of optimal flow rate of injected working agent. Usually, "Method of representative curved well" is applied, based on building the dependence of well production rate on flow rate of injected gas (Fig. 2). This method is well described in the literature ([4, 13] etc). It needs periodical field investigation of the well under different fixed duties, connected with material and human resources. Besides, the efficiency of optimum duty determination here is extremely low due to some reasons [4, 6, 8, 11, 13]. For example, is the change of hydrodynamic conditions within period, when well investigation is conducted, the effect of neighboring wells has not been considered. With this aim in [4] expressions have been proposed for bed fluid inflow to the bottom-hole and a system of differential expressions for motion of gas and gas liquid mixtures within well as well as methods for gaslift process forecast. However, implementation of this method shows, that the application of differential expressions to forecast gas liquid mixture flow within central pipe, being proposed in the [4], is difficult because of complexity of real gas-hydrodynamic conditions determination in the point of gas drive (here, in the lift shoe) at definite value of gas intake, which is indeed boundary condition for $x = L$ for the expression of GLM flow.

Besides, determinations of several parameters in the expressions are practically impossible, it leads to additional complication while application of expressions for GLM flow, proposed in [4]. To remove this problem it is necessary to develop a full mathematical model of the well-bed system considering GLM formation area. It will allow one to forecast the behavior of the process, taking place in the area of GLM formation. Thus, mathematical simulation in the area where injected gas mixes with formation fluids is very difficult. Above mentioned requires to apply non-standard approach to determine the function on the border $x = L$, where L is the coordinates of the inlet point of injected gas - tube shoe in this case (Fig. 1).

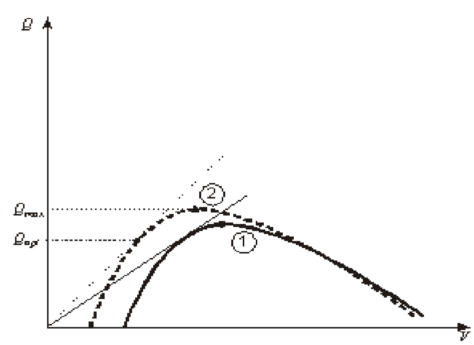

Fig. 2. Dependences of well flow rate Q on injected gas discharge V. 1- current tube diameter and submergence depth; 2- increased tube diameter and submergence depth.

2. Influence of submergence

It is known there exist a lot of parameters, affecting gaslift process, but the diameter d of the pipe and the coefficient of relative submergence $\overline{\varepsilon}$ play a principle role, while formation of dependence character *Q(V)* at the definite static level value h_{st} [10, 13]. It is important, that indicated factors' of the influence is similar for all the wells.

Relative submergence coefficient $\overline{\varepsilon}$ is determined as $\overline{\varepsilon} = \frac{l}{L}$, where l is submergence depth (see Fig. 1) and $0 < \overline{\varepsilon} \leq 1$, depending on static level value h_{st}.

According to results of work [13], along with the tubing diameter increase, the height of the *Q(V)* curve increases, i.e. production rate of the well increases which corresponds to optimum and maximum conditions of qaslift, but the increase of submergence "moves" *Q(V)* curve towards the origin (Fig. 2, curve 2).

So, we can take into consideration that gaslift well liquid production rate Q_j at specified static level is the function of the injected gas flow rate V, diameter d and relative submergence depth $\overline{\varepsilon}$: *Q=f(V, d, $\overline{\varepsilon}$)*. This dependence is similar for all the wells by other similar conditions.

In [2,9] the parabolic approximation is applied for the approximation of characteristic curves. Taking it into account the dependence *Q=f(V, d, $\overline{\varepsilon}$)* the following polynomial [7, 12] may be accepted

$$Q_j = K_e - (a_1 V^2 + b_1 V + c_1) \cdot K_d. \tag{1}$$

Coefficients K_e and K_d we will preliminary accept in the linear form (further it proves to be true results of field researches)

$$K_e = a_2 \overline{\varepsilon} + b_2,$$

$$K_d = a_3 d + b_3,$$

where coefficients $a_1, b_1, c_1, a_2, b_2, a_3, b_3$ have to be determined by the method of least squares [7].

Expression (1) describes liquid flow rate dependence at the wellhead Q_j (i.e. $x = o$) on the gas injection flow rate at definite values of pipe diameter and its submergence depth.

It is established that at lifting of a liquid its part can be flown down back downwards [8]. When dynamic level of the liquid strongly falls, this phenomenon is proved distinctly.

Therefore, values of flow rates of liquid at the wellhead and the tube shoe during the same moment may difer. Because of it expression (1) on $x = L$ can be used taking into account the corresponding proportionality coefficient *Kj*:

$$Q_{jL} = K_j \cdot \left[K_e - (a_1 V^2 + b_1 V + c_1) \cdot K_d \right], \tag{2}$$

Here proportionality factor K_j denotes the influence of many factors, as walls hydraulic resistance, and is experimentally determined. But it should be noted, that this phenomenon is temporary. That means, the values of liquid flow rate at tube shoe and wellhead during steady-state conditions will be similar and the value of proportionality factor K_j will be equal to one.

Complex field study were conducted for definition of factors $a_1, b_1, c_1, a_2, b_2, a_3, b_3$ and investigation of *Q=f(V, d, $\overline{\varepsilon}$)* dependence character.

3. Mathematical model of the well-bed system

Our aim is to find the general lows of inference of the parameters to gaslift process and generalize expression (2) for all the wells. Here, the motion of injected gas within annulus and the rising flow of GLM within central pipe are described by the following system of differential equation [4]

$$-\frac{\partial P}{\partial x} = \frac{1}{F}\frac{\partial Q}{\partial t} + \frac{2a}{F}Q \tag{3}$$

$$-\frac{\partial P}{\partial t} = \frac{c^2}{F}\frac{\partial Q}{\partial x}. \tag{4}$$

It should be noted, the area of GLM formation has definite volume. But it is impossible to forecast the coordinated of the noted area. Therefore, for simplification it is assumed that GLM formation takes place in the interval of (*L-0*) and (*L+0*). In this case the system of equations (1)-(2) should be considered with the following boundary conditions

at $0 \le x \le (L-0)$:

Q= Qz(t,x);

at $(L-0) \le x \le (L+0)$:

Q=Qz + Qr

at $(L+0) \le x \le 2L$:

Q= Qj+ Qz .

On the other hand, it is obvious that here the condition $Q_r = Q_j$ is fulfilled, where Q_r is the flow rate of oil and saturated gas, flowing from the bed to the bottom-hole and is determined by the following system of equations [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{p\beta\gamma_q f_q(1-\rho_n)}{Z(p)p_{at}\mu_q(p)}+\frac{S(p)f_n(\rho_n)}{a(p)\mu_n(p)}\right]k(p)\frac{\partial p}{\partial r}\right\}=-\frac{\partial}{\partial t}\left\{\left[\frac{p\beta\gamma_q(1-\rho_n)}{Z(p)p_{at}}+\frac{S(p)}{a(p)}\rho_n\right]m(p)\right\} \quad (5)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\left[\frac{f_n(\rho_n)}{a(p)\mu_n(p)}\right]k(p)\frac{\partial p}{\partial r}\right\}=-\frac{\partial}{\partial t}\left\{\left[\frac{\rho_n}{a(p)}\right]m(p)\right\}. \quad (6)$$

Solution procedure of this system is detail described in [1, 4].

Therefore the problem is to determine coefficients $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ of expression (1) using the results of the field researches. Here, offshore field "Neft Dashlari" has been selected, where about 450 wells are being operated by gaslift method.

The investigations were carried out in the well No.2390, which drains the X horizon. It has the following current technical data and parameters:

(1) *Well construction: double-row*
(2) *Well depth, m: 620*
(3) *Diameter of production casing, inches: 5*
(4) *Diameter of the I row, inches: 2.5*
(5) *Diameter of the II row, inches: 1.5*
(6) *Formation pressure, MPa: 4.65*
(7) *Static level, m: 550*
(8) *Wellhead pressure, MPa: 1.1*
(9) *Length of the lift, m: I row - 604; II row - 392*
(10) *Current oil output, t/day: 2.0*
(11) *Current water output, t/day: 11.0*
(12) *Gas factor*, $m^3/t : 320$

Under the carried out investigations the dependence of well production rate on injected gas flow rate has been determined at wider ranges of the central pipe length values, i.e. coefficient of relative submergence, and pipe diameters.

Here we consider only part of the obtained results on the influence the submergence of the second tube row (lift pipe).

The first tests show(Fig. 3) the possibility of choice of the well conditions, when the maximum production rate of well 2390 will be 5.4 t/day. It is 2.7 times more than production rate of well under the current conditions.

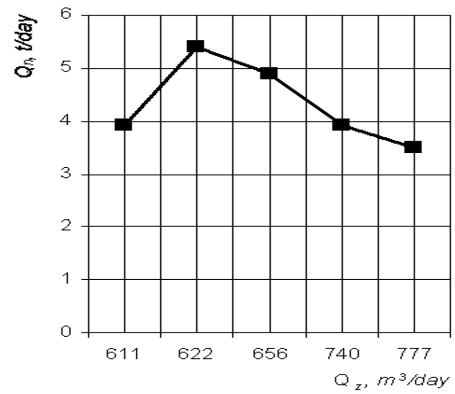

Fig. 3. Characteristically curve of well 2390.

The investigations were done for the following lengths of the central pipe: 316, 345, 372 and 410 m. Depths of submergence were 246m, 275m, 302m, 340m correspondingly. To make possible the comparison of Q(V) curves for different submergence depths, they have been drawn at the same coordinate system. Here, conventional (number of measurement) values for injected gas flow rate were used as abscissa, because gas flow rate values for each case do not present interest at the moment (Fig. 4). As we see the increase of submergence depth naturally leads to increase of well productivity. However, the fact, that the increase of submergence depth by 22.9% leads to recovery enhance by 60.5%, attracts our attention. This fact shows the sensitivity of gas lift process to the considered parameter.

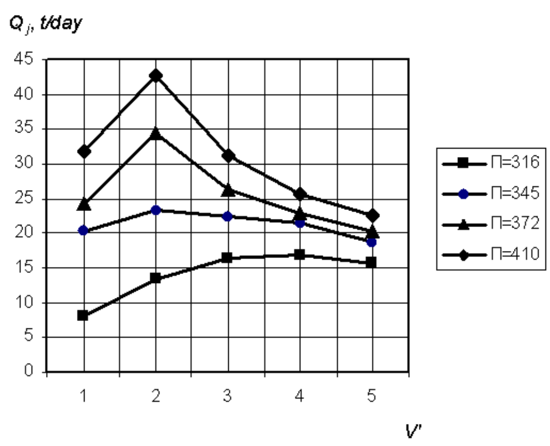

Fig. 4. . Characteristically curve of well 2390 at submergence depth 246 m, 275 m, 302 m, 340 m.

The dependence of maximum production rate of well on value of submergence for liquid (oil + water) and for oil separately (Fig. 5) has been found to determine the law of submergence influence to the process.

One can see in both cases the dependence is linear. Actually, if take into consideration that total rate of well is $Q_{ob} = Q_z + Q_n + GQ_n + Q_v$ then, for the liquid production rate of well and saturated gas we can write: $Q = vF + Q_z$, where G is gas factor, F is central pipe cross-section area. As F is a constant value, then it is obvious, that Q at definite value of F will depend only on the liquid flow rate and reservoir gas. In this case Q_z is also a constant value. On the other hand, as in the plot there are presented maximum values of well production rate for each submergence depth values, we can easily see, that the dependence between these values will be straight linear.

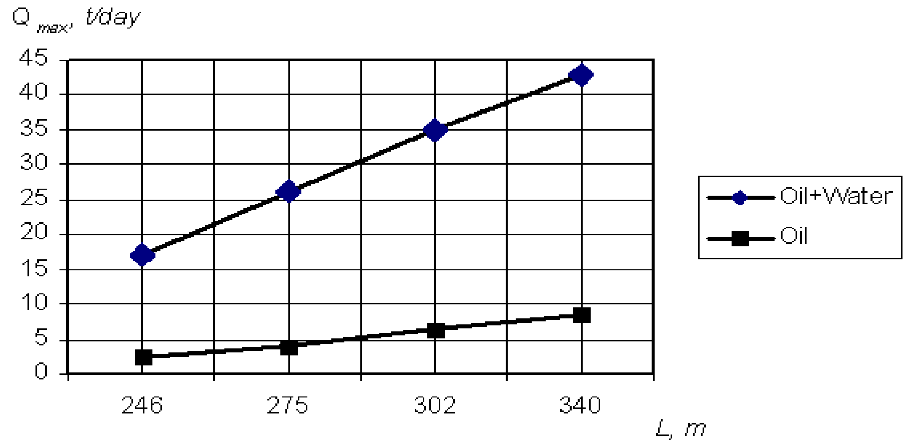

Fig. 5. Dependence of maximum well flow rate Q on submergence depth for liquid (oil+water) and for oil.

Submergence length, m	Gas flow rate Q_z, m^3/day	Max.oil production rate, t/day	Max. Liquid production rate, t/day
246	1373	2.5	16.9
275	1294	4.1	23.3
302	1061	6.3	42.7
340	778	4.6	42.8

Table 1. Maximum well production rates for oil and for liquid at different submergence and injected gas flow rate.

Data, given in the Table1, describe the relation between values of maximum rates for different submergence and gas flow rates, providing maximum production rates, corresponding this submergence (Fig. 6).

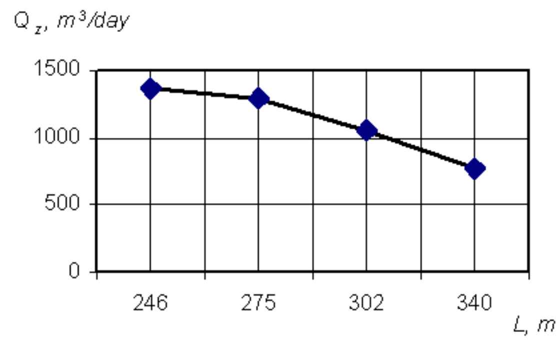

Fig. 6. Dependence of gas discharge, providing maximum rate on sinking value.

The curve, corresponding to these data demonstrates the character of dependence of injected gas rate, providing maximum oil production rate of well on submergence value (Fig. 6).

Therefore, the proposed approach allows one to consider the availability of a starting valves and to implement results obtained in the work [4] for solution of forecasting and control problems of the gaslift process [3, 5].

4. Conclusions

Given in the present work results, allows one to define character and degree of influence of depth of submergence on the qaslift process. It is established that dependence of the maximum value production rate of well on the submergence depth is nonlinear. It confirms results of research the works [10] spent in vitro (short tube). Therefore, the formula for K_e justifies itself.

Besides, it is necessary to notice that to develop the mathematical model of system a well-bad it is necessary to consider influence of length of submergence.

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